C2M11
Ratio Test

The ratio test is one of the most important tools in the study of infinite series. Its validity is a consequence of what we know about geometric series. For Maple purposes we will define the sequence upon which the series is based as a function of n. So we will use $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} a(n)$ which emphasizes that $a_n = a(n)$ is really a function of n.

Example: Discuss the convergence/divergence of the series $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{n\,5^n}$.

We use the ratio test and consider $\frac{a_{n+1}}{a_n} = \frac{2^{2n+3}}{(n+1)\,5^{n+1}} \cdot \frac{n\,5^n}{2^{2n+1}} = \frac{2^2\,n}{5\,(n+1)}$. Take the limit $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2^2\,n}{5\,(n+1)} = \frac{4}{5}$ and conclude that the given series converges because the limit is less than one. Now, let's do this same problem using Maple. Note how we define $a_n = a(n)$ **as a function**, but the ratio, r_n, **is an expression**.

Maple Example:

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> with(student):
> a:=n->2^(2*n+1)/(n*5^n);
```
nth term of series

$$a := n \to \frac{2^{(2n+1)}}{n\,5^n}$$

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> rn:=a(n+1)/a(n);
```
ratio for series

$$rn := \frac{2^{(2n+3)}\,n\,5^n}{(n+1)\,5^{(n+1)}\,2^{(2n+1)}}$$

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> rn:=simplify(rn);
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$$rn := \frac{4}{5}\,\frac{n}{n+1}$$

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> limit(rn,n=infinity);
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$$\frac{4}{5}$$

C2M11 Problems Use Maple to assist with the ratio test for the given series. Remember to include a concluding remark about the ratio test results.

1. $\sum_{n=1}^{\infty} \frac{(n+1)^2}{3^n\,n!}$
2. $\sum_{n=1}^{\infty} \frac{(3n)!}{2^{2n}7^n(n!)^3}$
3. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$